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PERIODICALLY VARIABLE TWO-TERMINAL IMPEDANCE DESCRIPTION AND MEASURING METHODS

The subject of this paper is the problem how to describe and measure impedance of a linear periodically time-varying (LPTV) two-terminal. If the two-terminal parameters change synchronously to the supply the two-terminal is able to reach a periodical steady state, but supplied with a sinusoidal source produces a non-sinusoidal voltage or current. The LPTV system in the steady state may be described with a circular parametric operator (CPO). Within the discrete time domain, this operator takes the form of a real entries matrix.

The circular parametric operator may be transformed into the spectral circular parametric operator (SCPO) using a two-dimensional DFT. This version of description makes it possible to assess quantitatively the LPTV system coercion and response harmonics aliasing. This paper presents two methods of determination of the impedance CPO of the LPTV two-terminal based on current and voltage signal measurements. The first method is designed for operating in the frequency domain, the second one operates in the discrete time domain and takes advantage of a genuine identification algorithm. Some results of computer simulations are presented.

Keywords: periodically time-varying two-terminal, circular parametric operator, measurement methods

1. INTRODUCTION

The presence of non-linear and non-stationary components of the power system cause distortion of current and voltage waveforms. Together with development of methods and technology of distortion compensation, a new challenge is to measure quantities connected with the phenomenon of creation of distortion. One of them is measuring the distorting two-terminal impedance or admittance. The time-varying circuits, e.g. switching converters or thyristor-driven loads, and the non-linear system components, e.g. transformers, saturated chocking coils or diode-equipped loads change their parameters periodically, synchronously with the power voltage. It seems that a linear periodically time-varying (LPTV) model is the best way of power system impedance description. The LPTV power system model may be introduced as a reasonable approximation of the non-linear reality. A physically supported LPTV model is easier to identify than a non-linear one and may be used as well for modeling, as for the description of real systems [1].

The frequency-domain description, analysis and interpretation of the linear time invariant (LTI) systems is a common practice. The steady state of the LTI two-terminal supplied with a sinusoidal voltage or current may be described by use of one complex number, i.e. the impedance or the admittance. There are many methods of complex impedance measurement. The situation is different in the case of the LPTV two-terminals which can also reach a periodical steady state, but supplied with a sinusoidal source produce non-sinusoidal voltage or current. It is impossible to describe such a phenomenon by means of one complex number. A description of the LPTV system is much more complicated.

The ways of description of LPTV systems used in automatics, multirate signal processing and telecommunication may be found in many publications. Some common representations of the LPTV systems including linear time invariant models obtained by blocking, linear switched time-varying (LSTV) systems and alias-component matrices are described in [2] and the references therein. Subspace model identification algorithms that allow the identification of a linear time-varying (LTV) state space model from an ensemble set of input-output measurements is presented in [3]. Discrete time LPTV systems also may be modeled by discrete time wavelets. It makes the system identification to be robust to narrow-band or impulse noise [4]. Description of the LPTV system as a time invariant block system is presented in [5]. These mathematical characterizations of the LPTV system are universal but their use is not simple.

In this paper a two-terminal is treated as a single input – single output (SISO) system in the manner presented in Fig. 1.

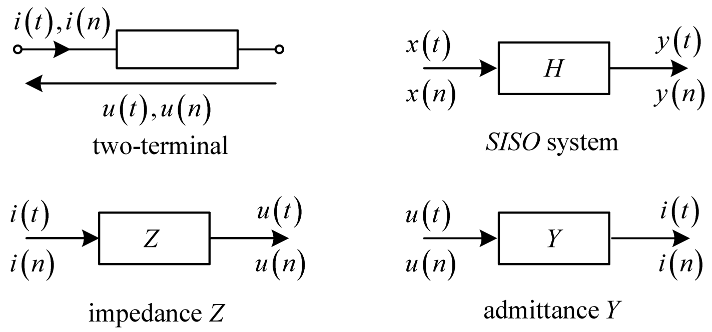

Fig. 1. A two-terminal treated as a SISO system.

The relationship between voltage and current of the LPTV two-terminal in the steady state can be described by use of the so-called circular parametric operator (CPO). It is assumed that the period of the system parameter changes is equal to the period of voltage and current changes. This assumption is usually met in the case of a power system and makes the description of the variable two-terminals simple and convenient. In the discrete time domain the CPO describes the relationship between vectors of one period samples of the input and output signals and takes the form of a real entries matrix. It is a generalization of the discrete circular convolution designed for the LTI systems. The discrete time description is more useful for application in practice. For the first time this way of system description was introduced by Siwczyński to realize a new numerical operator method for nonlinear systems analysis described in [6].

A spectral circular parametric operator (SCPO) which is a complex entries matrix describes the relation between the input and output harmonics phasors of the LPTV system. It is a generalization of a commonly used frequency characteristic of an LTI system. It may be obtained from the CPO by use of a two-dimensional discrete Fourier transformation.

The specific way of the variable two-terminal impedance description requires working out some measuring methods. In this paper two methods are presented. The first one is designed to operate in the frequency domain and makes it possible to determine entries of the SCPO on the basis of the measurements using sinusoidal coercion signals. The other method is based on an original identification algorithm which makes use of an ensemble set of the input-output measurements given in the form of the vectors of voltage and current samples.

The measurement error analysis and the influence of noise on the measurement accuracy are not the subject of this paper. It is assumed that all signals are noise-free, coherently sampled with an adequately high sampling frequency.

2. DESCRIPTION OF THE SISO LPTV SYSTEM

The relation of the input signal $x(t)$ to the output signal $y(t)$ for a SISO LTV system (Fig. 1) may be described with a differential equation of variable coefficients

$$\sum_{i=0}^{q} a_i(t) y^{(i)}(t) = \sum_{i=0}^{q} b_i(t) x^{(i)}(t). \tag{1}$$

The Equation (1) may be solved with the integral operator H

$$y(t) = Hx(t) = \int_{-\infty}^{\infty} h(t,\tau) x(\tau) d\tau. \tag{2}$$

The operator kernel $h(t, \tau)$ is the response to Dirac's pulse $\delta(t-\tau)$. For a time-varying system, it is a function of two variables, i.e. it depends on the current time t and on the moment of the pulse application τ.

Within the domain of discrete time, the relation of the input signal $x(n)$ to the output signal $y(n)$ for a SISI LTV system can be described by a difference equation of variable coefficients

$$\sum_{i=0}^{q} A_i(n)y(n-i) = \sum_{i=0}^{q} B_i(n)x(n-i). \tag{3}$$

Equation (3) can be solved with the operator given in the form of the following sum

$$y(n) = Hx(n) = \sum_{m=-\infty}^{\infty} h(n,m)x(m). \tag{4}$$

The operator kernel $h(n, m)$ is the response to Kronecker's pulse $\delta(n-m)$. In the case of a time-varying system, it is a function of two variables, i.e. it depends on the current time n, and on the moment the pulse has been applied m.
The N-periodical signal, $x(n+N) = x(n)$ can be expressed using Poisson's formula [7]

$$x(n) = \sum_{p=-\infty}^{\infty} \widehat{x}(n+pN), \tag{5}$$

where the so-called segment

$$\widehat{x}(n) = \begin{cases} x(n) & \text{for} \quad 0 \leqslant n \leqslant N-1 \\ 0 & \text{for} \quad \text{other} \quad n \end{cases}. \tag{6}$$

Applying the operator of the network (4) to the N-periodical input signal the response may be determined using the following formula [7]:

$$\begin{aligned} y(n) = Hx(n) &= \sum_{m=-\infty}^{\infty} h(n,m) \sum_{p=-\infty}^{\infty} \widehat{x}(m+pN) = \\ &= \sum_{m=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} h(n,m)\,\widehat{x}(m+pN) = \sum_{m'=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} h(n,m'-pN)\,\widehat{x}(m') = , \\ &= \sum_{m'=0}^{N-1} \sum_{p=-\infty}^{\infty} h(n,m'-pN)\,x(m') = \sum_{m'=0}^{N-1} \tilde{h}(n,m')\,x(m') \end{aligned} \tag{7}$$

where the circular pulse response is defined as

$$\tilde{h}(n,m) \equiv \sum_{p=-\infty}^{\infty} h(n,m-pN). \tag{8}$$

If the system is N-periodically variable, the pulse response satisfies the property

$$h(n+N, m+N) = h(n,m). \tag{9}$$

If the period of the system parameters changes, N is equal to the input signal period, the circular pulse response is N-periodical in n, i.e.

$$\tilde{h}(n+N, m) = \sum_{p=-\infty}^{\infty} h(n+N, m-pN) = \sum_{p=-\infty}^{\infty} h(n, m-(p+1)N) = \tilde{h}(n,m). \tag{10}$$

In this case the response $y(n)$ is N-periodical as well, i.e.

$$y(n+N) = \sum_{m=0}^{N-1} \tilde{h}(n+N, m)\, x(m) = \sum_{m=0}^{N-1} \tilde{h}(n,m)\, x(m) = y(n). \tag{11}$$

The N-periodical response of the system of N-periodically variable parameters to the N-periodical coercion can be determined with use of the so-called circular parametric operator

$$y(n) = \tilde{H}x(n) = \sum_{m=0}^{N-1} \tilde{h}(n,m)\, x(m), \qquad n = 0,\ 1,\ ...,\ N-1. \tag{12}$$

The circular parametric operator can be given in the form of the following matrix [7]

$$\begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \tilde{h}_{0,0} & \tilde{h}_{0,1} & \dots & \tilde{h}_{0,N-1} \\ \tilde{h}_{1,0} & \tilde{h}_{1,1} & \dots & \tilde{h}_{1,N-1} \\ \dots & \dots & & \dots \\ \tilde{h}_{N-1,0} & \tilde{h}_{N-1,1} & \dots & \tilde{h}_{N-1,N-1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_{N-1} \end{bmatrix}, \tag{13}$$

where $x_k = x(k)$, $y_k = y(k)$, $\tilde{h}_{n,m} = \tilde{h}(n,m)$. Or, shorter:

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{x}, \tag{14}$$

where $\mathbf{x}$, $\mathbf{y}$ are vectors of samples of one period of the coercion and response signals, and $\tilde{\mathbf{H}}$ is the circular parametric operator (circular parametric matrix).

For an LTI system, the pulse response is the one-variable function

$$h(n,m) \to h(n-m), \tag{15}$$

Then, the circular parametric matrix introduced in Eq. (13) passes to the form of the Toeplitz (circular) matrix

$$\begin{bmatrix} y_0 \\ y_1 \\ ... \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \tilde{h}_0 & \tilde{h}_{N-1} & ... & \tilde{h}_1 \\ \tilde{h}_1 & \tilde{h}_0 & ... & \tilde{h}_2 \\ ... & ... & & ... \\ \tilde{h}_{N-1} & \tilde{h}_{N-2} & ... & \tilde{h}_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ ... \\ x_{N-1} \end{bmatrix}. \tag{16}$$

This is a circular discrete convolution given in the matrix form.

The relation between the input and output harmonics phasors of the LPTV system is described by a spectral circular parametric operator (SCPO). To obtain the SCPO the relation between the input and output signals (14) has to be transferred into the frequency domain. An N-periodic signal may be expressed by means of the Fourier series of harmonic phasors. The analysis and synthesis equations can be described using a Fourier matrix, i.e.

$$\mathbf{x} = \frac{1}{2}\mathbf{F}\bar{\mathbf{X}}, \tag{17}$$

$$\bar{\mathbf{X}} = 2\mathbf{F}^{-1}\mathbf{x}, \tag{18}$$

where $\mathbf{x}$ is a vector of samples of one period of the N-periodic signal, $\bar{\mathbf{X}}$ is a vector of harmonic phasors and adjoint harmonics phasors, and the double value for the constant component, $\mathbf{F}$ is the Fourier matrix defined as

$$[\mathbf{F}]_{n,m} = \exp\left(\tfrac{2\pi}{N}nm\right), \quad n = 0, 1, ..., N-1, \quad m = 0, 1, ..., N-1. \tag{19}$$

To determine the frequency domain version of the CPO, i.e. the SCPO, one should multiply both sides of (14) by $2\mathbf{F}^{-1}$ and substitute (17). Taking into account (18) with reference to the signal $\mathbf{y}$, it yields

$$\bar{\mathbf{Y}} = \mathbf{F}^{-1}\tilde{\mathbf{H}}\mathbf{F}\bar{\mathbf{X}}. \tag{20}$$

Then, the spectral circular parametric operator takes the following form [7]

$$\tilde{\mathbf{H}}\left(k_y, k_x\right) = \mathbf{F}^{-1}\tilde{\mathbf{H}}\mathbf{F}, \tag{21}$$

where k_x and k_y are the harmonic enumeration numbers of the coercion and response signals.

The SCPO $\tilde{\mathbf{H}}\left(k_y, k_x\right)$ describes the relation between the input and output harmonic phasors of the LPTV system. This matrix obtained in the other way, not based on the circular parametric matrix, is called the alias component matrix in [8] and the frequency-response matrix for the LPTV system in [9]. The SCPO is a complex entries matrix. The vectors $\bar{\mathbf{X}}$ and $\bar{\mathbf{Y}}$ have a typical structure of the signal spectrum, their complex entries fulfils the relation

$$\bar{\mathbf{X}}(n) = \bar{\mathbf{X}}^*(N-n), \quad \bar{\mathbf{Y}}(n) = \bar{\mathbf{Y}}^*(N-n), \tag{22}$$

where a^* is a complex adjoint number to the a.
This is the reason why the matrix $\tilde{\mathbf{H}}\left(k_y, k_x\right)$ is pseudo symmetrical. Each element of the SCPO matrix has his equivalent adjoint element placed symmetrically according to the center of the matrix [10, 11]

$$\begin{aligned} \tilde{\mathbf{H}}\left(k_y, k_x\right) &= \tilde{\mathbf{H}}^*\left(N-k_y, N-k_x\right) \\ \tilde{\mathbf{H}}(0, k_x) &= \tilde{\mathbf{H}}^*(0, N-k_x) \qquad \text{for} \quad k_x,\ k_y = 1,\ \ldots,\ N-1. \\ \tilde{\mathbf{H}}\left(k_y, 0\right) &= \tilde{\mathbf{H}}^*\left(N-k_y, 0\right) \end{aligned} \tag{23}$$

The SCPO creates the possibility for the quantitative assessment of aliasing the input and output harmonics. Entries of the SCPO matrix lying outside the main diagonal qualify the manner of transformation of coercion harmonics into other harmonics of system response. The l-th line of the spectrum matrix includes entries decisive in the manner of processing all coercion harmonics into the l-th harmonic of system response. The k-th column together with the $(N-k)$-th column of the spectrum operator describe in which way the k-th harmonic of the coercion is processed into all response harmonics. Each harmonic phasor of the response signal is dependent on each coercion harmonic through two complex coefficients multiplied by the harmonic phasor and the adjoint harmonic phasor, respectively. This is the way of expression of the LPTV system time dependence described by the SCPO. That is why it is impossible to describe an LPTV system using a single one-dimensional frequency response typical for the LTI system.

For an LTI system, the matrix $\tilde{\mathbf{H}}$ is circular (16) and after its transformation into the spectrum domain according to (21), the matrix $\tilde{\mathbf{H}}\left(k_y, k_x\right)$ takes the form of a diagonal matrix. In this case the "separation" of harmonics, typical for the LTI system, is clearly visible. Diagonal entries of this matrix make up the frequency response of the LTI system.

3. THE LPTV TWO-TERMINAL IMMITANCE MEASUREMENT

The result of an LTI two-terminal impedance measurement is one or two real numbers, i.e. module and phase. The sinusoidal current coercion of the LPTV two-terminal produces the non-sinusoidal (polyharmonic) voltage response. The impedance of the LPTV two-terminal for a given frequency of sinusoidal current coercion must be defined as a group of numbers, e.g. one column of the impedance SCPO. Determination of the LPTV two-terminal impedance operator takes the form of an identification process based on the measurements.

3.1. The frequency domain method

To determine the q-th column of the impedance SCPO some actions must be realized according to the following algorithm.

1. The two-terminal should be supplied two times with two sinusoidal currents of given frequency, of two different phases (needed data: current phasors I_{q1} and I_{q2}).
2. Each time the voltage signals should be measured (obtained data: one period voltage sample vectors $u_1(n)$ and $u_2(n)$).
3. The voltage harmonic phasors should be determined using DFT (obtained data: voltage harmonic phasors U_{p1} and U_{p2} for $p = 0, 1, ..., \frac{N}{2}$).
4. The SCPO entries may be determined by solving $N/2$ complex linear equations of the form

$$\begin{cases} I_{q1}\tilde{\mathbf{Z}}_{p,q} + I_{q1}^{*}\tilde{\mathbf{Z}}_{N-p,q}^{*} = U_{p1} \\ I_{q2}\tilde{\mathbf{Z}}_{p,q} + I_{q2}^{*}\tilde{\mathbf{Z}}_{N-p,q}^{*} = U_{p2} \end{cases} \quad \text{for} \quad p = 1, 2, ..., \frac{N}{2} \tag{24}$$

and for the direct component, of the form

$$\begin{cases} I_{q1}\tilde{\mathbf{Z}}_{0,q} + I_{q1}^{*}\tilde{\mathbf{Z}}_{0,N-q}^{*} = U_{01} \\ I_{q2}\tilde{\mathbf{Z}}_{0,q} + I_{q2}^{*}\tilde{\mathbf{Z}}_{0,N-q}^{*} = U_{02} \end{cases}. \tag{25}$$

To determine the whole SCPO the above-mentioned algorithm has to be repeated for columns number $q = 0, 1, ..., N/2$. The other half of the operator is determined using the relationship (23).

3.2. The time domain method

Determination of the LPTV two-terminal impedance CPO sometimes cannot be based on the sinusoidal coercions, e.g. the power system impedance measurement [12, 13]. In this case the following identification algorithm operating in the discrete time domain can be used.

The circular parametric operator may be determined on the basis of the set of K measured coercion signals $\mathbf{X}$ and the set of K related response signals $\mathbf{Y}$. The following relation implied by (14) is used here:

$$\tilde{\mathbf{H}}\mathbf{X} = \mathbf{Y}, \tag{26}$$

where $\tilde{\mathbf{H}}$ is the sought for circular parametric operator, $\mathbf{X}$ is a matrix of K input signals, each column is a vector of one input signal samples, and $\mathbf{Y}$ is a matrix of K system response signals.

In the case where $K = N$, i.e. the number of coercion and response signals K equals to the size N of the square circular parametric matrix, the problem has an unequivocal solution:

$$\tilde{\mathbf{H}} = \mathbf{Y}\mathbf{X}^{-1}, \tag{27}$$

which depends on the condition meaning the linear independence of the coercion signals

$$\det \mathbf{X} \neq 0. \tag{28}$$

In the case where $K < N$, the matrix Eq. (26) has an infinite number of solutions. The optimal solution should be chosen. One can propose to seek for an operator which describes a system with the smoothest changes of parameters. With consideration for (26), the optimization problem may be originally defined as

$$\left(\Delta\tilde{\mathbf{h}}_n\right)^T \Delta\tilde{\mathbf{h}}_n \to \min, \tag{29}$$

$$\mathbf{X}^T\tilde{\mathbf{h}}_n = \mathbf{y}_n, \tag{30}$$

where $\tilde{\mathbf{h}}_n = \left[\begin{array}{cccc} \tilde{h}_{n,0} & \tilde{h}_{n,1} & \ldots & \tilde{h}_{n,N-1} \end{array}\right]^T$ is a vector of entries of the n-th line of matrix $\tilde{\mathbf{H}}$, and $\mathbf{y}_n = \left[\begin{array}{cccc} y_{n,0} & y_{n,1} & \ldots & y_{n,K-1} \end{array}\right]^T$ is a vector of n-th samples of all K response signals (entries of the n-th line of matrix $\mathbf{Y}$). The vector of increments of $\tilde{\mathbf{H}}$ entries for the n-th line is defined as

$$\Delta\tilde{\mathbf{h}}_n = \left[\begin{array}{cccc} \Delta\tilde{h}_{n,0} & \Delta\tilde{h}_{n,1} & \ldots & \Delta\tilde{h}_{n,N-1} \end{array}\right]^T, \tag{31}$$

$$\Delta\tilde{h}_{n,m} = \tilde{h}_{n,m} - \tilde{h}_{n(-)1,m(-)1}, \tag{32}$$

where (-) is a subtraction mark of modulo N. The vector $\Delta\mathbf{h}_n$ may be also defined with the use of a circular unit delay matrix of the form

$$\mathbf{P}_1 = \begin{bmatrix} 0 & \ldots & 0 & 1 \\ 1 & \ldots & 0 & 0 \\ \ldots & & \ldots & \ldots \\ 0 & \ldots & 1 & 0 \end{bmatrix}, \tag{33}$$

i.e.

$$\Delta\tilde{\mathbf{h}}_n = \tilde{\mathbf{h}}_n - \mathbf{P}_1\tilde{\mathbf{h}}_{n-1}. \tag{34}$$

The criterion (29) means minimization of increases of matrix $\tilde{\mathbf{H}}$ coefficients in the direction of the main diagonal. Such criterion choice originates from the fact that in case of a linear time invariant system (LTI) the relationship between signals of the coercion and response is described in the periodic steady state by means of the circular matrix (16). Then, the increases of entries defined by means of (32) are equal to zero. A choice of the criterion (29) means the research of the circular parametric operator describing the system of the least variability of parameters, realizing (26). The Equation (30) results from (26).

The optimizing problem given by (29) and (30) may be solved using Lagrange method in the manner given by the author of [6, 7, 11]. The method is described in [11, 14, 15]. The result is the iterative solution

$$\tilde{\mathbf{h}}_n = \left(\mathbf{1} - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\right)\mathbf{P}_1\tilde{\mathbf{h}}_{n-1} + \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{y}_n, \tag{35}$$

where $\mathbf{1}$ is a unit matrix of a proper dimension.

In order to effect the iterations, the invertibility of the matrix $(\mathbf{X}^T\mathbf{X})$ is necessary, thus, the following condition must be fulfilled

$$\det\left(\mathbf{X}^T\mathbf{X}\right) \neq 0. \tag{36}$$

This requires linear independence of the coercion signals included in the matrix $\mathbf{X}$.

The obtained identification algorithm (35) has a standard form of a discrete state equation

$$v(n+1) = \mathbf{A}v(n) + \mathbf{B}u(n), \tag{37}$$

where

$$\mathbf{A} = \left(\mathbf{1} - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\right)\mathbf{P}_1, \tag{38}$$

$$\mathbf{B} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}. \tag{39}$$

The convergence of the iterations (35) depends on the eigenvalues of the matrix $\mathbf{A}$. The construction of the matrix $\mathbf{A}$ (38) turns its eigenvalues to be

$$|\lambda_i| \leqslant 1 \quad i = 1, 2, ..., N, \tag{40}$$

independently of the shape of the signals included in the matrix $\mathbf{X}$ [16]. In the situation when $|\lambda_i| = 1$, the solution of a homogeneous system is of a periodic signal form. To avoid this unwanted component of the solution to appear, the required form of the initial vector is, $x(0) = [0, 0, ..., 0]^T$, i.e. for the iterations (35) it is

$$\tilde{\mathbf{h}}_0 = [0, 0, ..., 0]^T, \tag{41}$$

The eigenvalues of the matrix $\mathbf{A}$ (38), of the absolute value $|\lambda_i| = 1$ take the form

$$\lambda_{k,N-k} = e^{\pm j\frac{2\pi}{N}k} \tag{42}$$

and appear if none of the periodic signals in the matrix **X** include the k-th harmonic [16]. If any $|\lambda_i| = 1$ then the solution of the non-homogeneous system (37) may aim at infinite amplitude. It is possible if the vector

$$r(n) = \mathbf{B}u(n) = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{y}_n, \tag{43}$$

which is the element of $v(n+1)$ (i.e. for iterations (35) – the element of $\tilde{\mathbf{h}}_n$), includes at least one harmonic of the angular frequency equal to the eigenvalue of the matrix **A** of the (42) form. On the other hand, the vector $r(n)$ (43) is a linear combination of the signals located in **X** and does not include the k-th harmonic of the angular frequency corresponding to the eigenvalues of the (42) form. This means that the identification algorithm (35) is not divergent and is convergent to an N-periodical solution [16].

Identification of the circular parametric operator consists of iterative determination of the matrix lines with reference to the previous lines. Each line of the matrix $\tilde{\mathbf{H}}$ obtained from the iteration fulfils Eq. (30) and the optimizing criterion (29). The iterations should be realized until they obtain the N-periodical steady state. It means that for each element of the line $\tilde{\mathbf{h}}_n$ after N iterations nearly the same result is obtained with specified accuracy

$$\left|\tilde{h}_{m+N,n} - \tilde{h}_{m,n}\right| \leqslant \varepsilon \quad n = 0, 1, ...N-1, \tag{44}$$

where ε is a satisfactorily small deviation value.

The suitably specified one period of the iterations makes up the determined CPO.

4. NUMERICAL SIMULATIONS

Some numerical experiments were carried out to check the correctness of operation and properties of the presented methods. By use of a discrete simulation of an analog system the CPO of a simple two-terminal was determined. It will be named the "original" one. The diagram of the two-terminal and its parameters periodic changes functions are presented in Fig. 2. Graphs of the LPTV two-terminal impedance CPO $\tilde{\mathbf{Z}}(n,m)$ and of the SCPO magnitude $\left|\tilde{\mathbf{Z}}(k_U,k_I)\right|$ are presented in Fig. 3.

These two presented methods were tested and compared by determining the SCPO column. Two sinusoidal coercion current signals of different phase were used. The response voltage signals were obtained by operating the original CPO on the coercion signals. Using the identification algorithm the identified impedance CPO was determined and using 2D DFT the identified SCPO was calculated. They are presented in Fig. 4. The obtained SCPO includes only two non-zero columns related to the coercion frequency. The identification algorithm input data carried only this information. The same two SCPO columns were determined using the frequency domain method as well. Characteristics of the methods' error obtained by using clear and disturbed sinusoidal coercion signals are presented in Fig. 5.

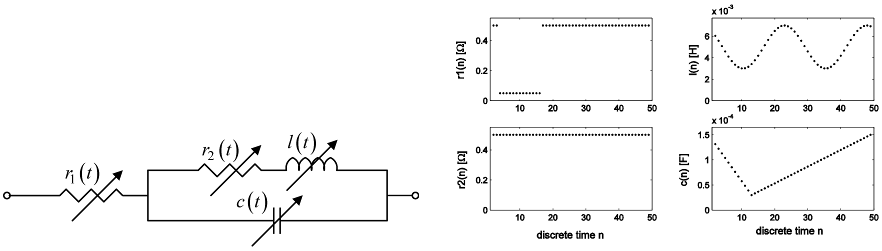

Fig. 2. Diagram of the LPTV two-terminal that is used as an example and the functions of periodic changes of the elements parameters.

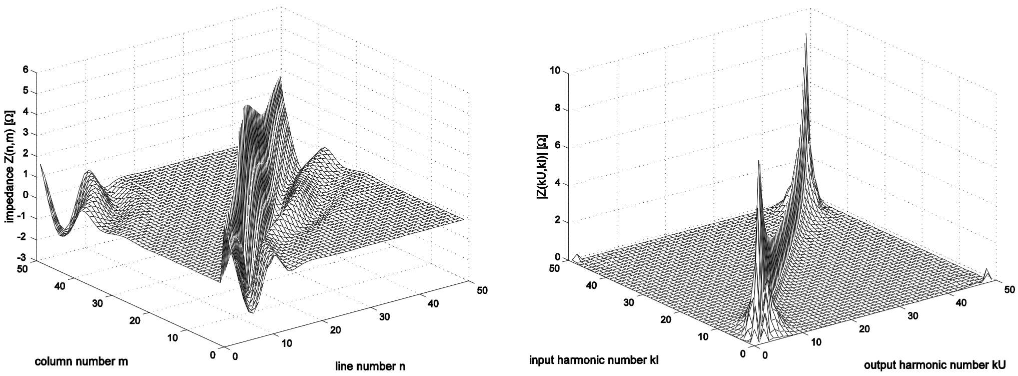

Fig. 3. Graph of the "original" impedance circular parametric operator $\tilde{\mathbf{Z}}(n,m)$ and the "original" spectral circular parametric operator magnitude $\left|\tilde{\mathbf{Z}}(k_U,k_I)\right|$ of the LPTV two-terminal.

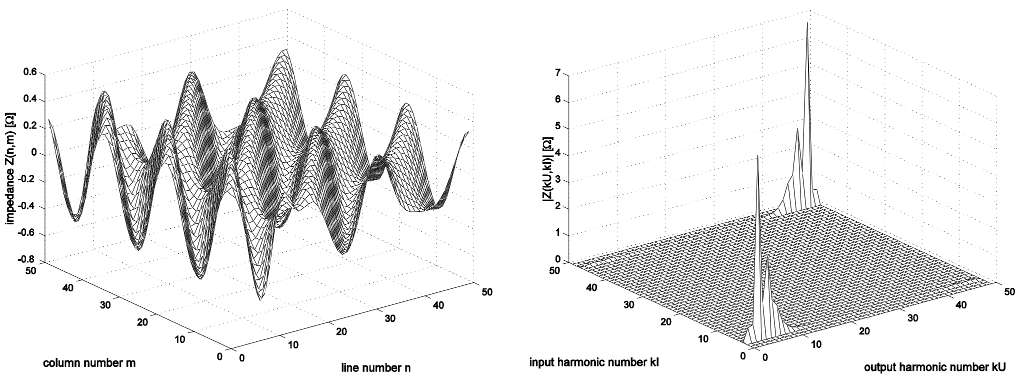

Fig. 4. Graph of the impedance circular parametric operator $\tilde{\mathbf{Z}}(n,m)$ and the spectral circular parametric operator magnitude $\left|\tilde{\mathbf{Z}}(k_U,k_I)\right|$ obtained from the identification based on two sinusoidal coercion signals.

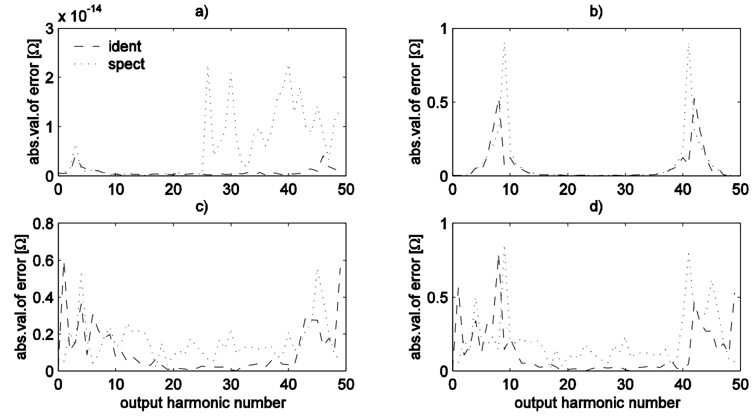

Fig. 5. Characteristics of methods error of one column of the SCPO determination using presented methods based on the sinusoidal coercion signals: a) non-disturbed, b) disturbed with a harmonic signal of different frequency, c) disturbed with a noise (random) signal, d) disturbed with a different frequency harmonic signal and with a noise (random) signal (ident – identification algorithm, spect – spectral domain determination).

The identification algorithm operating in the time domain can be used for the CPO determination on the basis of N-periodic signals of any shape. The only restriction is the linear independence of the used coercion signals. Some CPOs and SCPOs obtained as the result of the identification algorithm based on the different numbers of noise (random) signals are presented in Fig. 6. Results of these operators' operation on the periodic rectangular impulse and the sinusoidal signals are shown in Fig. 7. The response voltage signals obtained by operation of the original operator and the identified operator on the test coercion signals are compared.

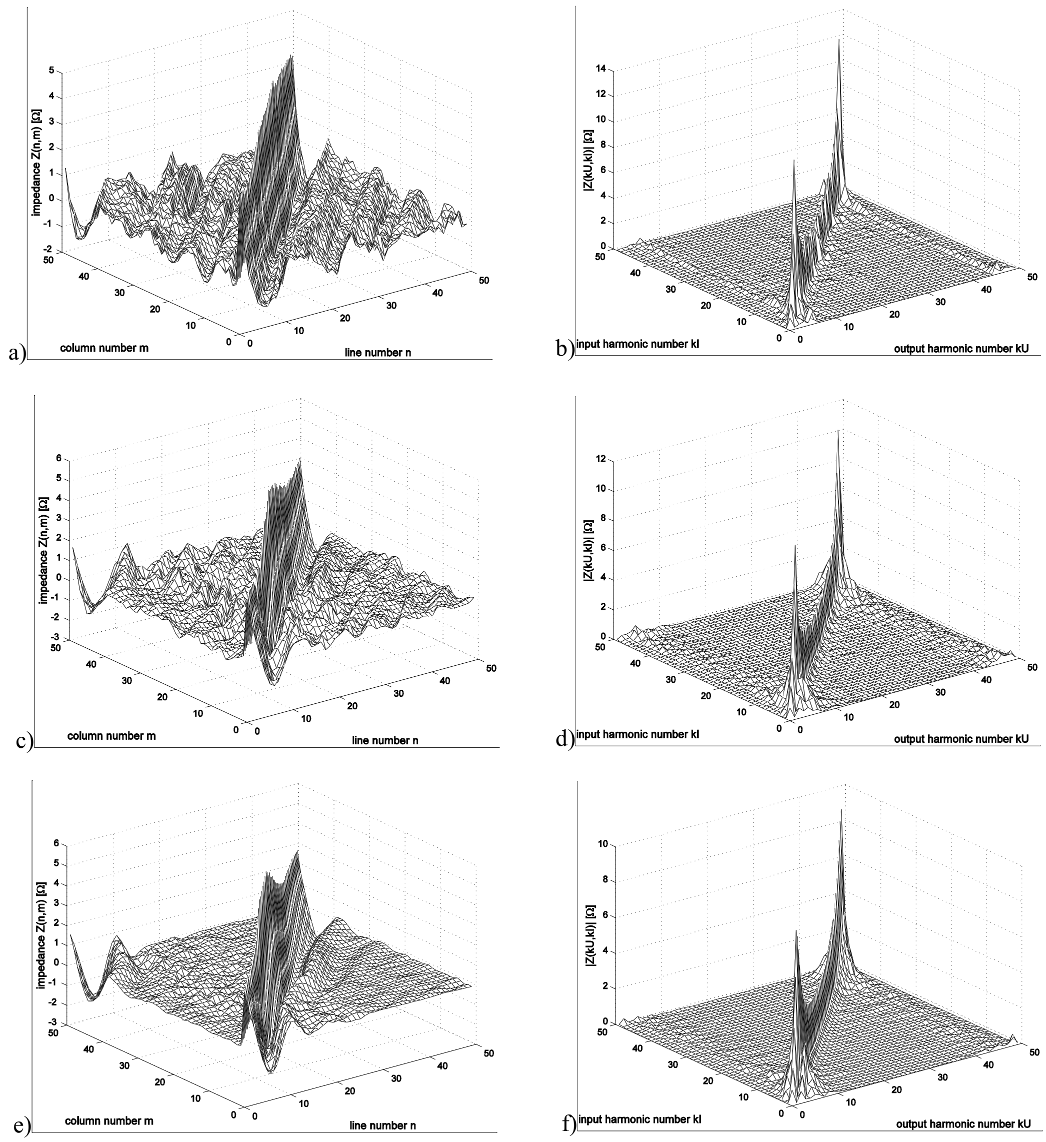

Fig. 6. The CPOs (graphs a), c) and e)) and SCPOs (graphs b), d) and f)) obtained as the results of the identification algorithm based on the different numbers of noise (random) signals: a), b) – 3 signals; c), d) – 10 signals; e), f) – 30 signals. The dimension of operators is 50×50.

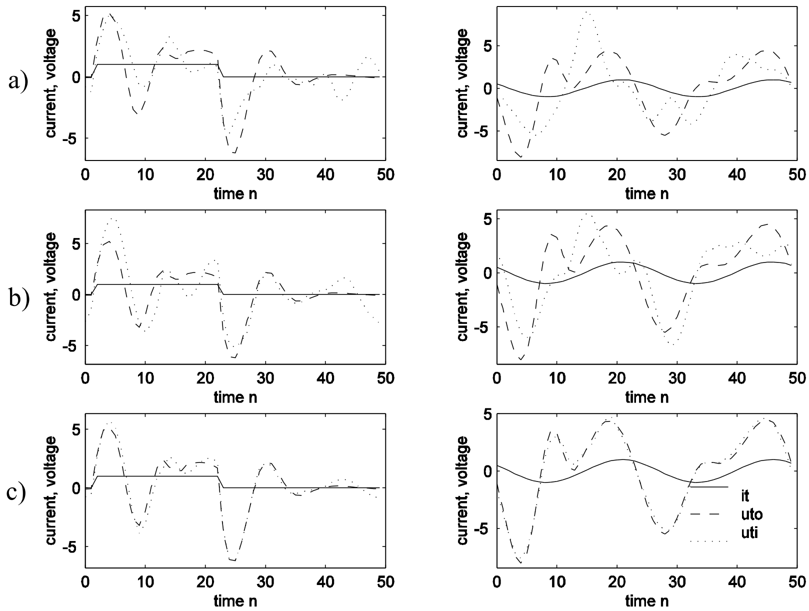

Fig. 7. Comparison of the response voltage signals obtained by operating of the original operator (symbol “uto”) and of the identified operator (symbol “uti”) on the test coercion signals (symbol “it”). Identification based on the different numbers of noise (random) signals: a) – 3 signals; b) – 10 signals; c) – 30 signals.

5. CONCLUSIONS

The use of the circular parametric operator to describe the relation between terminal signals of the periodically time-varying two-terminal network seems to be a good practical solution. Within the domain of discrete time, such operator takes the form of a real element matrix. It is very convenient if discrete signal processing may be used. The spectral circular parametric operator gives the quantitative assessment of aliasing of the input and output harmonics, the phenomenon characteristic for time-varying and non-linear systems. Two methods of the LPTV two-terminal impedance CPO determination based on the current and voltage signals measurement were presented. The frequency domain method requires sinusoidal coercion and can be used for the determination of the LPTV two-terminal impedance for a selected frequency. The time domain method based on the genuine identification algorithm may be used for determination of the whole impedance CPO. It brings the expected results only if the identification data i.e. the current and voltage measured signals include the necessary portion of the information about the performance of the two-terminal being identified.

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